Journal of Civil Engineering and Architecture 13 (2019) 195-203
doi: 10.17265/1934-7359/2019.03.004

Discharge Coefficient Measurements Using Heron's Fountain

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Abstract: Civil Engineering design students at CSUN (California State University, Northridge), aimed to demonstrate the pneumatic action of liquid water as it flows through an airtight one-way vessel system which is known as Heron's Fountain. This project explores hydraulic and pneumatics principles commonly found in environment control systems, such as the non-isothermal heating facilities located on the CSUN campus. Since this was a simply constructed version of an ancient Greek fountain, its development required the collaboration of the team to execute its simple function. The parameters involved were diameter, length, height, and density. This analysis utilizes Pascal and Bernoulli's equations to reinforce the principles of fluid mechanics. The fountain action is described based on flow rate and head loss is described by Darcy's equation. Friction loss with an angled fitting attached to the fountain head is described by Reynold's equation. The experiment observed the performances of two types of reentrant tube fittings for head loss: straight and angled. The experiment enhanced the educational experience of the research team by bringing together creative ideas from different educational and cultural backgrounds. The results of the experiment concluded with a 0.58% error for the straight fitting and 5.3% error for an angled fitting.

Key words: Heron's Fountain, hydraulic principles, air pressure, pneumatics, friction factor, engineering education.

1. Introduction

Heron's Fountain begins in antiquity with the Greek mathematician and engineer, Heron of Alexandria. He invented mechanical devices powered by air, water and steam which were used for a variety of reasons. He enjoyed using his inventions for educational purposes and taught pneumatic principles by how his devices worked. Heron's Fountain is an apparatus which responds when liquid water is added to the fountain basin and generates a water jet from the fountain head. This instrument operates using pneumatic principles and the principles of non-isothermal flow [1].

Pneumatic systems are commonly found in dams, transmission systems and power stations. Teaching the principles of hydraulics to aspiring engineers can be challenging but rewarding in the end [2]. Heron's Fountain is a device used to teach and demonstrate the principles of hydraulics with water and air pressure. Many scientists have dreams of perpetual motion machines that do not dissipate energy. This fountain is not continuous and therefore it is not perpetual since it only works for a certain amount of time until the velocity becomes zero [3]. The interesting device is a demonstration of hydrostatic pressure. Heron's Fountain is the first model to incorporate both automatic recharging energy and flow-triggering of the fountain [4]. Unfortunately, the fountain has several setbacks that can only be re-energized by some manual transfer of liquid which can cause liquid spillage. The team constructed the project by using plastic tanks and vinyl tubing to obtain the flow rate of the water jet and compared it to the theoretical values derived from Pascal's and Bernoulli's equations [5, 6]. Comparisons of the results were made and studied after various trials. The objective of this project is to enhance the knowledge about the principles of hydraulics in a

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simple manner and to provide a better understanding of air pressure using this educational design.

The inventor, Heron of Alexandria sold his inventions to wealthy patrons who enjoyed the splendor of his mechanical inventions as centerpieces for grand occasions. Fig. 1 illustrates the original invention called "A Satyr Pouring Water from a Wine-skin into a full Washing Basin, without making the contents overflow" also known as Heron's Fountain.

2. Physical Model Details and Testing

The research team gathered reusable materials that allowed better visibility of the liquid as it runs throughout the system. The materials chosen to achieve best results were three 1.3-gallon containers with plastic lids, vinyl tubing and adhesive shown in Table 1. It is best to construct Heron's Fountain as air-tight as possible in order to achieve best results.

2.1 Construction Procedures

Three containers were aligned in a stacked configuration. Three pieces of vinyl tubing of different diameters were cut into the appropriate lengths. Holes were drilled through the lids to pass the tubing through. With the holes aligned, the water-resistant adhesive was applied to secure the assembly and create an air-tight vessel. Silicone adhesive was also applied on the lid threads and sealed with adhesive tape. Fig. 2 illustrates the placement of the parts. As indicated in

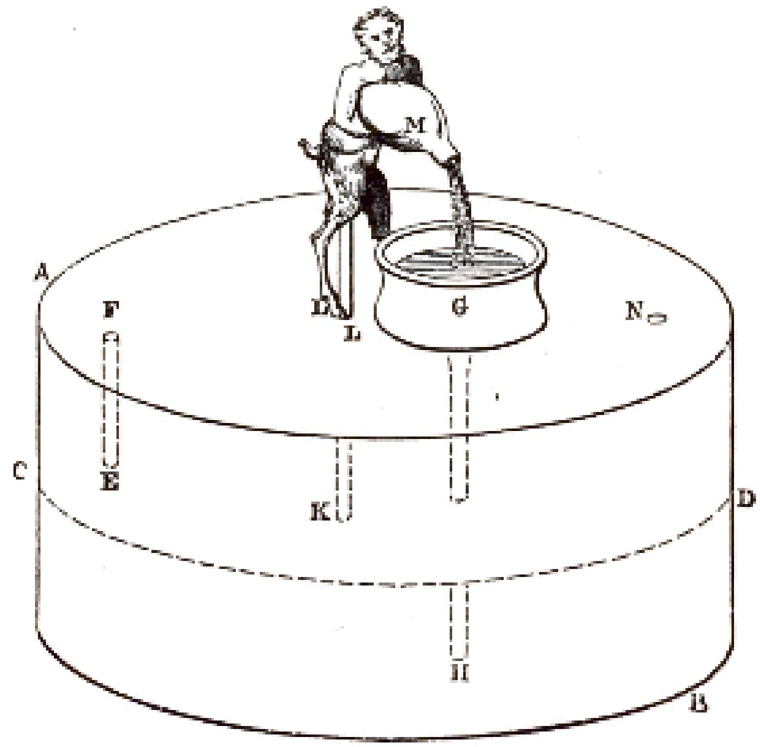

Fig. 1 Original drawing of Heron's Fountain [3].

Table 1 Orifice fittings and various vinyl tubing sizes.

Count	Dimension	Material/Items
1	½"	Straight fitting
1	½"	90° fitting
1	¾" - 24.1" (Tube #1)	Clear vinyl tube
1	¼" - 6.50" (Tube #2)	Clear vinyl tube
1	½" - 11.8" (Tube #3)	Clear vinyl tube

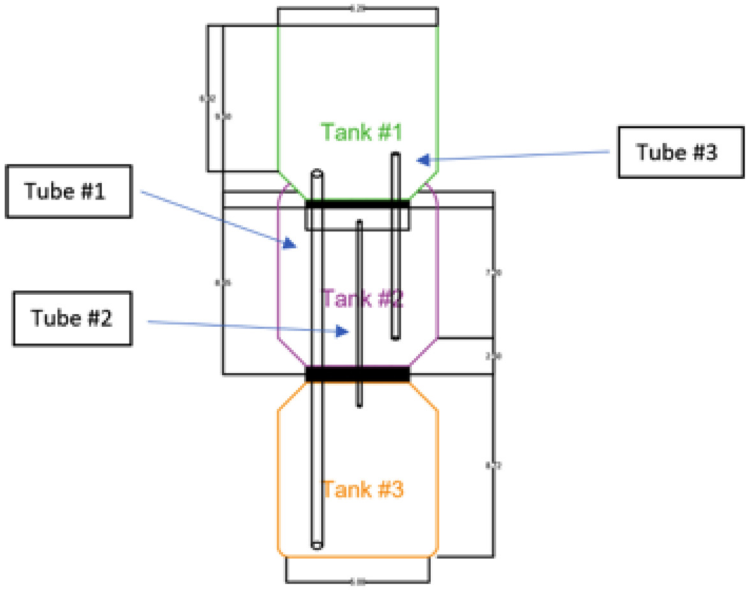

Fig. 2 Project drawing of a custom-made Heron's Fountain based on original construction.

the drawing, Tank 1 is open to the atmosphere while the other tanks are closed. Tank 3 carries the stream into the basin. The fountain process was initiated by pouring water into the basin. Fig. 7, provided in the Appendix, shows the final constructed fountain.

2.2 Test Procedures

Tank 2 was filled with one gallon of warm water funneled through Tube 3. Warm water will increase internal pressure in Tank 2 as water drains into Tank 3. The rising water level inside Tank 3 generates internal air pressure, forcing air through Tube 2 into Tank 2. As Tank 2 increases in internal air pressure, this pressure forces the reserve water through Tube 3 up into the basin of Tank 1. Tube 3 is the point of the experimental control where the two fittings are attached. The hydraulic pressure is created as water enters Tank 1 and increased air pressure in Tank 3. Tank 2 creates a rising jet of water through an orifice attached to Tube 3 in Tank 1. The performance tests occur at this point with the two fittings by testing the flow rate, Q, of each orifice.

3. Theoretical Analysis

Simplicity is at the core design and construction. In the development of the project, the use of imagination and creative thinking were used to test the performance of a variety of orifices attached to the fountain's head as shown in Fig. 3. The air pressure, P_c, in Tank 2 was determined using Eq. (1), the moment air was forced into Tank 2. Before this movement occurs the velocity of the water-level is zero. The velocity V of the water jet was determined using Bernoulli's Eq. (2) in order to calculate the flow rate using Eq. (3).

$$P_c = g(\rho_w h_w + \rho_a h_a) \quad (1)$$

where, P_c is pressure inside the middle container, g is gravitational acceleration, ρ_w is water density, h_w is height of water life, ρ_a is air density, and h_a is height of air.

$$\frac{P}{\gamma} + \frac{V^2}{2g} + Z \quad (2)$$

where, γ is specific weight of fluid, V is velocity and Z is the datum height.

$$Q = VA \quad (3)$$

where, Q is flow rate and A is cross sectional area.

4. Discussion of Results

To find the flow rate between Tank 1 and Tank 2, Bernoulli's equation was used.

$$\frac{P_2}{\gamma_w} + \frac{V_2^2}{2g_c} + Z_2 = \frac{P_0}{\gamma_w} + \frac{V_0^2}{2g_c} + Z_0 + h_f \quad (4)$$

where, head losses = h_f = pipe loss + fittings losses and Pipe loss is described as:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \quad (5)$$

the expression $f\frac{L}{D} = K$ was used for the fittings. Therefore, K is classified for each type of fitting.

The general equation for head loss can be described as follows:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} + K_1\frac{V^2}{2g} + K_2\frac{V^2}{2g} + \cdots + K_n\frac{V^2}{2g} = \left(f\frac{L}{D} + \sum_{i=1}^{n} K_i\right)\frac{V^2}{2g} \quad (6)$$

The K factors represent the friction coefficients of each fitting and are shown in Table 2 [5].

These angled fittings are attached at the threated side which is indicated as the entrance, as shown in Fig. 3.

Eq. (4) is used between Tank 1 and Tank 3 to determine pressure at Tank 3, P_3. Because the cross-sectional area of Tank 1 is equal to the cross-sectional area of Tank 3, $V_0 = V_3$ therefore V_0 and V_3 cancel and create the following expression $P_0 = 0$.

$$P_3 = (Z_0 - Z_3)\gamma_w \quad (7)$$

Eq. (4) was applied between Tank 3 and Tank 2 where $V_2 = V_3$, therefore V_2 and V_3 cancel. This yields the following equation to solve for P_2:

$$P_3 - P_2 = (Z_2 - Z_3)\gamma_w \quad (8)$$

Pressure at each point is determined by applying Eq. (4) with the head losses to find the velocities in the pipes.

Eqs. (4) and (5) were applied to Tank 1 and Tank 2 to solve for V_{P3}. The theoretical flow rate of the straight fitting is $Q_3 = A_{P3} * V_{P3} = 0.0067\ [\frac{\text{ft}^3}{\text{sec}}]$. The experimental average flow rate concluded was Q_{exp} = 0.00705 [ft^3/sec]. The expression used to determine this percentage is:

$$\%\ error = \frac{Q_{exp} - Q_3}{Q_3} * 100 \quad (9)$$

This yields the percent error to be 5.3%.

The theoretical flow rate of the angle fitting is $Q_3 = A_{P3} * V_{P3} = 0.0058\ \left[\frac{\text{ft}^3}{\text{sec}}\right]$. The experimental average flow rate concluded was Q_{exp} = 0.00566 [ft^3/sec]. yielding to a percent error of 0.58%. Sample calculations of friction factor (f) are provided in the Appendix.

Table 2 Friction constant values of each reentrant fitting.

Experiment	K_1 Entrance	K_2 Elbow	K_3 Exit
#1 Straight fitting	0.78	-	1
#2 90° fitting	0.78	30	1

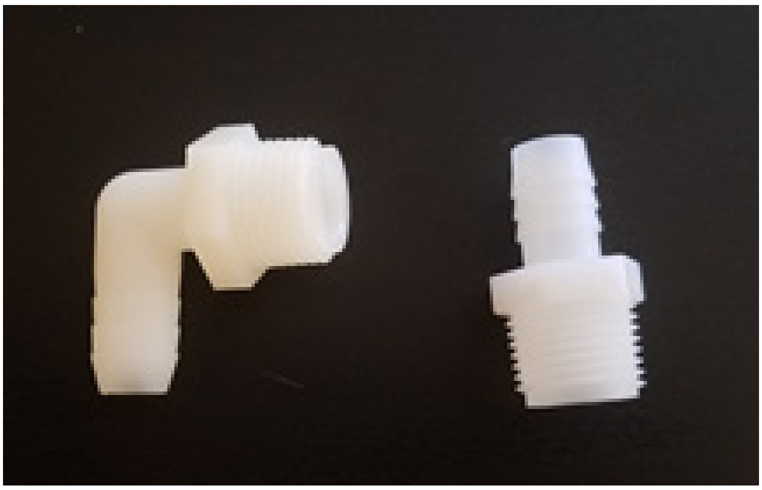

Fig. 3 Fittings: ½" - 90° (elbow) and ½" - 180° (straight).

The experimental research indicates the flow rates of angled and straight fittings at three separate trials. As shown in Figs. 4 and 5, the correlation among these flow rates per each trial for angled and straight fittings is 99% based on the R^2 values. Due to a lower friction factor in the straight fitting attachment, the flow rate through the system was more efficient.

The theoretical velocity for the straight fitting was 4.91 [ft/s] while the angled fitting was 4.23 [ft/s]. The experimental velocity for the straight fitting was 5.2 [ft/s] while the angled fitting was 4.29 [ft/s]. The velocity value in the angled fitting is less than the velocity in the straight fitting for both the experimental and the theoretical due to the head loss in the angled fitting. The research team came to the conclusion that the percentage error for the straight fitting and the angle fitting were due to the size of the tubes, since Tubes 2 and 3 are smaller in diameter compared to Tube 1. Minor leakage of air pressure tends to give such percentage of errors.

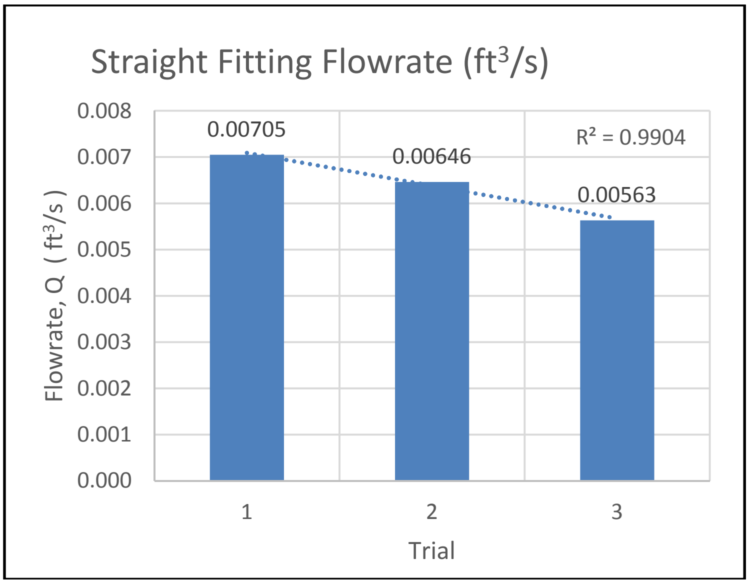

Fig. 4 Straight fitting flow rate per trial.

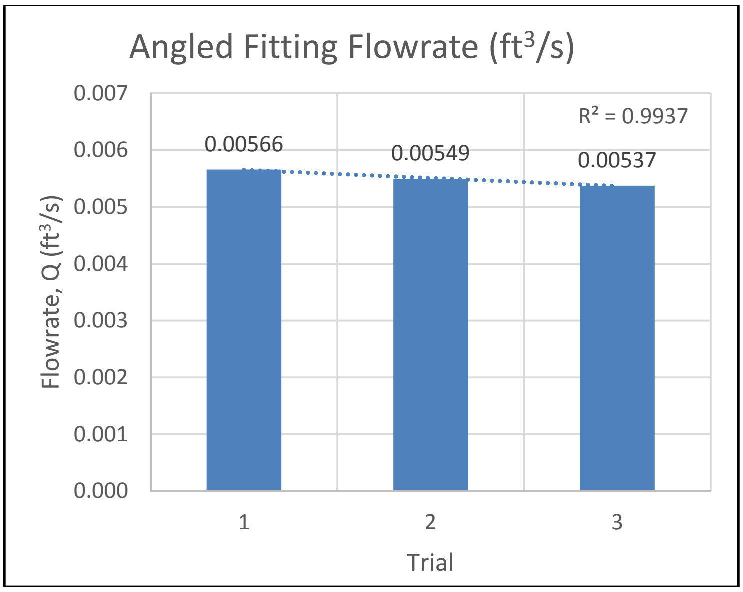

Fig. 5 Angled fitting flow rate per trial.

Fig. 6 Group member Francisco Villalobos demonstrates Hero's fountain at student youth STEM program.

5. Educational Objectives

Each member in the research team was able to understand basic hydraulic principles that this device teaches. With students having different backgrounds and different levels of knowledge about the subject, it was necessary to discuss and conduct research about the topic with one another throughout meetings. Learning about hydraulics in this simple manner will provide a better comprehension in future related classes and professional practice. Furthermore, the members were able to gain theoretical knowledge on how to apply Bernoulli's equation to a hydraulic mechanism in which enhances members' understanding about the analysis and properties of fluids.

Every member in the research team comes from a minority background, including women in engineering. Through teamwork and creativity, the students were able to bring this effort to light, despite different cultural backgrounds. This model could be used to cast attention on science and engineering to young high school students, as it can be explored by them physically. In addition, the members of the research team can serve as role models to younger minority students. This effort was demonstrated to the student youth at a Los Angeles STEM program as seen in Fig. 6, which carries on the tradition of hands on learning and teamwork.

During this demonstration, the children were able to have a hands-on learning experience by capitalizing on the fountains size.

This presentation included a brief presentation with slides of pictures, graphs and results. This collaborative effort utilized simple techniques and methods to conduct the experiment and produce results. The design and construction used a creative approach to test the flow rates of two types of orifice reentrant fittings attached to the fountain's head. The collaborative nature of the project brought purpose and meaning to the project while enriching the participation in the research of this ancient invention that was modeled for teaching purposes. Heron's invention was used as a basis to test the performance of the attached fittings.

6. Conclusion

In this educational research, Heron's Fountain was designed, constructed, tested, and theoretically analyzed. This project was conducted to find the flow rate of two different orifice fittings, including a straight orifice and an angled orifice. Many principles have been used in this project such as, the flow rate equation and the Bernoulli's, Darcy's, and Reynold's equations. Therefore, this research effort has enhanced the usage of such equations and engineering applications. Some challenges the research team experienced included, where to begin the construction, finding the pressure of the Tanks, and preventing leakage. The team was led by women in engineering and with their innovative ideas, the goal was achieved after discussing different approaches that could be applied to this project. Through intense research, the team was able to find a solution to finalize the Heron's Fountain project.

Acknowledgments

The student research team would like to express their sincere and profound appreciation for the support provided by Dr. Tadeh Zirakian and Dr. David Boyajian, Professors of Civil Engineering in the Department of Civil Engineering and Construction Management at California State University, Northridge, and also Marcial Martin, Engineer Advisor Process and Mechanical at STANTEC, Bakersfield, CA, USA.

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Appendix

Fig. 7 Final construction of Heron's Fountain. Based on the schematic diagram the fountain was constructed and tested until there was no leakage.

In addition, sample calculations for determination of the friction factor are also provided in the following. Two methods were used to solve for the friction factor for A. the straight fitting and B. the angle fitting.

A. Solving for the friction factor as a function of Reynold's Number

Considering $\epsilon = 0.0006\ in$ for a plastic pipe, we can get:

$$\frac{\epsilon}{D} = 0.0001$$

An initial approximation of Reynold's can be obtained by using the experimental velocity $V = 5.2\frac{ft}{sec}$

$$\nu = 0.0000092\ \frac{\text{ft}^2}{\text{sec}}\ @\ 80^0F$$

$$Re = \frac{DV}{\nu} = 23550 > 4000$$

Therefore, the flow is turbulent:

From the Moody Diagram with $\frac{\epsilon}{D} = 0.0001$ and $Re = 19300$, therefore is **f = 0.0278**

B. Solving for the friction factor using the experimental velocity

As a first approximation $V = 4.26 \frac{ft}{sec}$, so

$$Re = \frac{DV}{\nu} = 23550 > 4000$$

Therefore, the flow is turbulent:
From the Moody Diagram with $\frac{\epsilon}{D} = 0.0001$ and $Re = 19300$, therefore is $\boldsymbol{f}$ **= 0.028**.
Note that only in laminar flows does the following apply (Re < 2100): $f = \frac{64}{Re}$.